FLOOD ROUTING IN RIVERS BY MUSKINGUM'S METHOD WITH NEW ADJUSTED COEFFICIENTS

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ABSTRACT

For determining of Muskingum model coefficients, requires to an output hydrograph. Such hydrograph is not available in most rivers. In this research, the Muskingum's new coefficients are determined by the method that which is not require to output hydrograph and its accuracy is high. This coefficient is determined based on kinematic wave model with suitable scheme. The comparison between results of Muskingum model with new coefficients and dynamic wave model, showed that the new coefficients, are valid at special conditions. The new coefficients were adjusted by optimization technique for all conditions. The new adjusted coefficients are function of bed slop, bottom width and Manning's roughness coefficient for the river. The results of these coefficients were validated by dynamic wave model.

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1 INTRODUCTION

The flood flow in rivers is an unsteady flow and its characteristic is varied with time. These variations are made by human or natural factors. The flow variations are described by a hydrograph in hydrology. The flood routing investigates the variations of depth and discharge flow in rivers or channels. The methods or models of flood routing are different. The full dynamic model is the most accurate of them, in which the continuity and momentum equations are solved completely. Other methods such as kinematic wave, the continuity equation and summarized form of momentum equation are solved. These methods were compared by Samani and Shayannejad (Samani, 2000).
The kinematic wave method is valid if the local and convection accelerations are negligible and slopes of surface water and bed are same (Chaudhry, 1993). The generated error in results of kinematic wave model is due to basic assumptions and finite difference numerical solution (Weinmann, 1979). An usual and simple method for flood routing in rivers is Muskingum's method. This method was based on continuity equation and its equation is following:

$$O_2 = C_1 I_2 + C_2 O_1 + C_3 I_1 \tag{1}$$

where I_1 and I_2 =input discharges at t_1 و t_2 time steps ; O_1 و O_2 =output discharge at t_1 و t_2 time steps. C_1, C_2, C_3 =constant coefficients are which determined by a given input and output hydrograph.
The disadvantages of this method are:

1. It requires to an output hydrograph for calculating of its constant coefficients.
2. It determines an output hydrograph only at one point of river.
3. The applied assumptions in this method cause low its accuracy.

The many of researchers have presented coefficients for Muskingum's method to remove above disadvantages. For example Cunge (Cunge, 1965), Ponce (Ponce, 1986) and Bowen and Koussis (Bowen, 1989) presented a series of coefficients, but the accuracy was still low.

In this paper the Muskingum's method with new adjusted coefficients have been derived of kinematic wave and then they have been adjusted by full dynamic model. This model validates the result of the new method.

2 MATERIALS AND METHODS

The kinematic wave method is combination of continuity equation and an equation for flow resistance as Chezy-Manning equation. These equations are:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \tag{2}$$

$$A=K.Q^{B} \tag{3}$$

Where Q=discharge;. A=area cross-section; x=distance; t=time, $B=\frac{3}{5}$ and:

$$K=\left(\frac{nP^{\frac{2}{3}}}{\sqrt{S_0}}\right)^{\frac{3}{5}} \tag{4}$$

Where P=wetting perimeter; n=roughness Manning's coefficient; S_0=bed slope.
K in equation 4 is determined by considering a given discharge (base flow) and calculation of wetting perimeter for this discharge.
The derivative of equation 3 is:

$$\frac{\partial A}{\partial t}=K.B.Q^{B-1}\frac{\partial Q}{\partial t} \tag{5}$$

The combination of equations 2 and 5 is:

$$\frac{\partial Q}{\partial x}+K.B.Q^{B-1}\frac{\partial Q}{\partial t}=0 \tag{6}$$

For solving equation 6 by numerical method, its terms are discrete following form (Chow, 1988):

$$\frac{\partial Q}{\partial x}=\frac{Q_{i+1}^{J+1}-Q_{i}^{J+1}}{\Delta x} \tag{7}$$

$$\frac{\partial Q}{\partial t}=\frac{Q_{i+1}^{J+1}-Q_{i+1}^{J}}{\Delta t} \tag{8}$$

$$Q=\frac{Q_{i}^{J+1}+Q_{i+1}^{J}}{2} \tag{9}$$

Where i=local step number; J=time step number; Δt=time between two sequential time step; Δx=distance between two sequential local step.
By substituting equations 7, 8 and 9 into equation 6 gives following equation:

$$Q_{i+1}^{J+1}=C_1.Q_{i}^{J+1}+C_2 Q_{i+1}^{J} \tag{10}$$

Where:

$$C_1 = \frac{\Delta t}{\Delta t + K.B.C_0 \Delta x} \tag{11}$$

$$C_2 = \frac{K.B.C_0 \Delta x}{\Delta t + K.B.C_0 \Delta x} \tag{12}$$

$$C_0 = \left(\frac{Q_i^{J+1} + Q_{i+1}^{J}}{2} \right)^{B-1} \tag{13}$$

Equation 10 is Muskingum's method with new coefficients and a coefficient is equal to zero. In spite of old Muskingum's method, this new coefficients are not constant during time step calculations, because the coefficients depend on C_0 and it is not constant. Besides for calculating of the new coefficients do not require to a given output hydrograph and flood routing can be carried out at any point of river.

The grid size calculation must choose so that the Courant number is equal or less than one. This number is:

$$C = \frac{C_K . \Delta t}{\Delta x} \tag{14}$$

Where C =Courant number; C_K =celerity (velocity of wave transport). It is determined from equation 3:

$$C_K = \frac{\partial Q}{\partial A} = \frac{1}{K.B.Q^{B-1}} \tag{15}$$

The result of this new method is compared with the results of full dynamic model that its accuracy has been validated by real data. The equation 2 and following equation (momentum) constitute full dynamic model:

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} + g\left(S_f - S_0\right) = 0 \tag{16}$$

Where V =flow velocity; y =depth of flow; g =gravitational acceleration; S_f =slope of energy grade line.Equations 2 and 16 are Saint –Venant equation and are solved by numerical method. For this work, they are discrete by Preissmann scheme (Li, 1975). Then for each reach of river, two equations with four unknown variables are constituted. These variables are depth and velocity of flow at two ends of any reach. Thus for M reach (with M+1 nodes), 2M nonlinear equation with 2m+2 unknown variables are constituted. Thus two equations are required, that are gained from boundary conditions. For example, the downstream boundary condition is rating curve and upstream boundary condition is an input hydrograph. The nonlinear equation is linearized by Newton-Raphson method. Finally depth and velocity of flow (and then discharge) are determined at any node and any time.

In this paper, the coefficients of equation 10 were adjusted in order to increasing of the new method accuracy, by nonlinear optimization technique with using full dynamic model. Firstly, these coefficients were changed to following form:

$$C_1' = K_1 C_1 \tag{17}$$

$$C_2' = K_2 C_2 \tag{18}$$

$$C_0' = K_3 C_0 \tag{19}$$

Instead of C_1, C_2 and C_0 at equation 10, C_1', C_2' and C_0' were applied. Secondly K_1, K_2 and K_3 were determined by nonlinear optimization technique according to following objective function:

$$F = \sum_{t=1}^{N} \left(Qm_t - Qc_t\right)^2 \tag{20}$$

Where F =objective function; Qm =output discharge calculated by full dynamic wave; Qc =output discharge calculated by equation 10 with new adjusted coefficients (equation 17, 18 and 19) and N =number of time step.

Thirdly were determined relationship between K_1, K_2, K_3 and characteristic of river

3 RESULTS AND DISCUSION

Figure 1 shows the output hydrograph at a distance one kilometer, calculated by full dynamic model and Muskingum's method with new coefficients (C_1, C_2 and C_0) for a hypothetic input hydrograph and following input data:

n =0.035; S_0 =0.001; bottom width= 20m; $\Delta t = 1\min$; $\Delta x = 100m$

Figure 1 shows that there is different between results of two methods. In this research was concluded that with increasing of bed slop and decreasing of bottom width and slope of input hydrograph, the results of two methods were similar. On the other hand, in these cases, the kinematic wave is dominated.

For increasing of accuracy of equation 10 its coefficients were adjusted by optimization technique and values of K_1, K_2 and K_3 were determined. These coefficients were not constant and were depended on characteristic of river. K (Equation 4) was chosen as preventative of characteristic of river, because it depends on bed slope, wetting perimeter (depends on bottom width) and roughness Manning's coefficient.

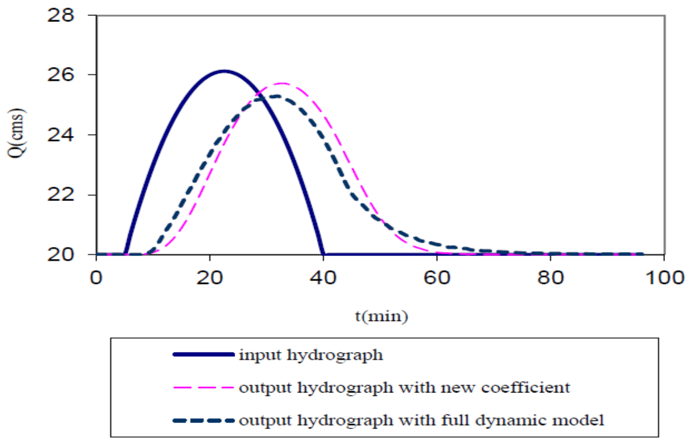

Figure 1: Comparison of results of full dynamic and Muskingum with new coefficients methods

Figure 2 shows the relationship between K_1, K_2, K_3 and K. The statistical analysis gives following equation:

$$K_1 = -0.0391 LnK + 1.0289 \tag{21}$$

$$K_2 = 0.0577 LnK + 0.956 \tag{22}$$

$$K_3 = -0.59961 LnK + 1.4705 \tag{23}$$

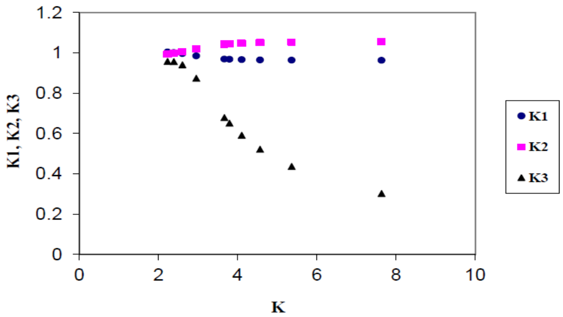

Figure 2: Variations of K_1 و K_2 و K_3 **related to** K

Figure 3 validates Muskingum's method with new adjusted coefficient. The following data were used in this figure are:

n =0.02; S_0 =0.003; bottom width= 30m; $\Delta t = 1\min$; $\Delta x = 100m$

The accuracy of the new adjusted coefficients is more than Cunnge, Ponce and Bowen ones. Figure 3 shows results of Cunge coefficients for example. Thus Muskingum's method with new adjusted coefficients is acceptable in rivers.

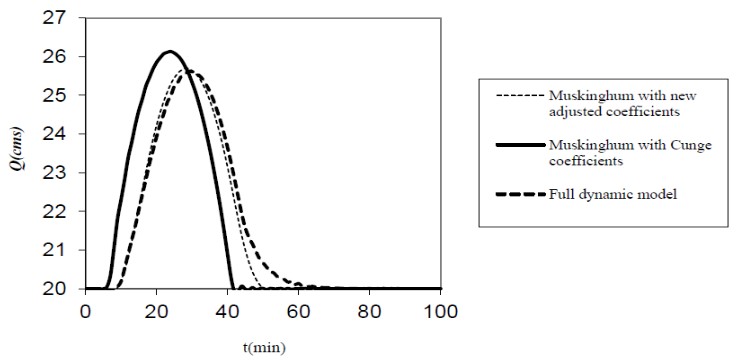

Figure 3: Comparison of output hydrograph by different methods

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